International Journal of Research in Advent Technology, Vol.6, No.12, December 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

Numerical Solution of Fuzzy Pure Multiple Retarded Delay Differential Equations

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Abstract-In this article, we propose the new Fuzzy Runge Kutta method of order four meant for Fuzzy multiple retarded delay differential equations and check whether the same method can be applied to solve Fuzzy pure multiple retarded delay differential equations. A numerical example with graphical illustrations is presented to verify the theory.
Index Terms-Pure Delay, Multiple Delay, Fuzzy Delay Differential Equations, Numerical Solution, Runge-Kuta Method.

1. INTRODUCTION

In modern Engineering and Technology, numerical methods play a vital role in evolution of problems involving power transmission, circuit design and analysis. Also in nature the role of numerical solutions is very useful since cyclone, rain or any natural predictions are always approximate in their accuracy with some error. The study of carbon dating is followed in any forensic research, DNA test to find similarities in genetic issues, and in almost all the fields of Genetic Engineering, medical science, etc., the role of numerical solutions is very important. In such a manner in this article we adopt numerical solution of fuzzy multiple retarded delay differential equations. Alfredo Bellen and Marino Zennaro studied numerical methods for delay differential equations in detail.[2]. S. Abbasbandy and T. Allahviranloo discussed numerical solution of fuzzy differential equation by Runge-Kutta method [1]. Guang Da Hu, Guang Di Hu and S. A Meguid discussed on Stability of Runge-Kutta methods for delay differential systems with multiple delays [4]. Baruh Cahlon and Darrell Schmidt analyzed the stability of systems of delay differential equations [3]. T. Jayakumar, A. Parivallal and D. Prasantha Bharathi solved fuzzy delay differential equations numerically by Runge-Kutta method [5]. In section 2, we present the concept of fuzzy multiple retarded delay differential equations, In section 3, we explain fuzzy pure multiple retarded delay differential equations. In section 4, we propose Runge –Kutta method of order four for solving fuzzy multiple retarded delay differential equations. In Section 5 a numerical example is studied to test the theory and the graphical illustrations are presented. We conclude the study of this paper in section 6.

2. FUZZY MULTIPLE RETARDED DELAY DIFFERENTIAL EQUATIONS (FMRDDE):

Let us consider the Initial value problem in ordinary differential equations,

$$\begin{cases} y'(t) &= f(t, y(t)) \\ y(0) &= y_0 \end{cases} \tag{1}$$

where $f:[0,\infty)\times R \to R$ is continuous. We would like to interpret (1) using Seikkala's derivative and $y_0 \in E^1$. Let $[y_0]^\alpha = [\underline{y}^\alpha(t), \overline{y^\alpha}(t)]$. By Zadeh's extension principle, we get $f:[0,\infty)\times E^1 \to E^1$, where ,

$$[f(t,y)]^\alpha = [\min\{f(t,u): u \in [\underline{y}^\alpha(t), \overline{y^\alpha}(t)]\}, \max\{f(t,u): u \in [\underline{y}^\alpha(t), \overline{y^\alpha}(t)]\}] \tag{2}$$

.

So from (2), the (1) became fuzzy differential equation. The concept of uncertainty and randomness arises whenever we deal with natural parameters so fuzzy is adopted in modeling any physical problems. Differential equations just used to study the rate of change but it is not possible to know when and how does the change occurred by ordinary differential equations.

So we adopt delay differential equation.. In delay differential equation we are evaluating the past with the present data. In nature, Let us consider a simple problem of life expectancy as an example, Life span depend upon so many management parameters like life style, mental stress, Allergy, chronic diseases , body cholesterol, Body Mass Index(BMI), etc.,. Each of these parameters produces rate of change in Expected life span but uncertainty is different people having same diseases, life style are not supposed to have same life span. Time is very important parameter then, because above people maybe at different ages. In such case multiple delay differential equations play a important role. The mathematical structure of multiple retarded delay differential equations is given by

$$\begin{cases} y'(t) = f(t, y(t), y(t-\tau_1), y(t-\tau_2), \\ \quad \dots, y(t-\tau_n)) & t_0 \le t \le t_n \\ y(t) = \phi(t) & -\tau \le t \le t_0 \\ y(t_0) = \phi(t_0) \end{cases} \tag{3}$$

Similar to that of (2), (3) follows as $f:[0,\infty)\times R\times R\times\ldots\times R\to R$ which is continuous and

$$[f(t,u(t),u(t-\tau_1),u(t-\tau_2),\dots,u(t-\tau_n)]^{\alpha} =$$

$$\begin{aligned}&[\min\{f(t,u(t),u(t-\tau_1),u(t-\tau_2),\dots,u(t-\tau_n))):\\ &u(t)\in[\underline{y}^{\alpha}(t),\overline{y^{\alpha}}(t)],\\ &u(t-\tau_1)\in[\underline{y}^{\alpha}(t-\tau_1),\overline{y^{\alpha}}(t-\tau_1)],\dots,\\ &u(t-\tau_n)\in[\underline{y}^{\alpha}(t-\tau_n),\overline{y^{\alpha}}((t-\tau_n))],\},\\ &\max\{f(t,u(t),u(t-\tau_1),u(t-\tau_2),\dots,u(t-\tau_n))):\\ &u(t)\in[\underline{y}^{\alpha}(t),\overline{y^{\alpha}}(t)],\\ &u(t-\tau_1)\in[\underline{y}^{\alpha}(t-\tau_1),\overline{y^{\alpha}}(t-\tau_1)],\dots,\\ &u(t-\tau_n)\in[\underline{y}^{\alpha}(t-\tau_n),\overline{y^{\alpha}}((t-\tau_n))],\}]\end{aligned} \tag{4}$$

3. FUZZY PURE MULTIPLE RETARDED DELAY DIFFERENTIAL EQUATION: (FPMRDDE)

The Pure Multiple retarded delay arises as special case of delay differential equation. The equation is so called because there is no *y(t)* and the derivatives involving delay terms [3]. Such a system is given by

$$\begin{cases} y'(t) = f(t, y(t-\tau_1), y(t-\tau_2),\dots, y(t-\tau_n)) & t_0 \le t \le t_n \\ y(t) = \phi(t) & -\tau \le t \le t_0 \\ y(t_0) = \phi(t_0) = \phi_0 \end{cases} \tag{5}$$

From (5), it follows that $f:[0,\infty)\times R\times R\times\ldots\times R\to R$ which is continuous and

$$[f(t,y(t-\tau_1),\dots,y(t-\tau_n))]^{\alpha} = \begin{aligned}&[\min\{f(t,u(t-\tau_1),\dots,u(t-\tau_n))):\\ &u(t-\tau_1)\in[\underline{y}^{\alpha}(t-\tau_1),\overline{y^{\alpha}}(t-\tau_1)],\dots,\\ &u(t-\tau_n)\in[\underline{y}^{\alpha}(t-\tau_n),\overline{y^{\alpha}}((t-\tau_n))],\},\\ &\max\{f(t,u(t-\tau_1),\dots,u(t-\tau_n))):\\ &u(t-\tau_1)\in[\underline{y}^{\alpha}(t-\tau_1),\overline{y^{\alpha}}(t-\tau_1)],\dots,\\ &u(t-\tau_n)\in[\underline{y}^{\alpha}(t-\tau_n),\overline{y^{\alpha}}((t-\tau_n))],\}]\end{aligned}$$

The FPMRDDE all the properties, definitions and tests, etc., of delay differential equations. Because of its initial value will be same as that of Retarded delay differential equations (1). The purpose of studying pure retarded delay differential equations is it directly gives the result of past state without the solution of present state.

4. FOURTH ORDER FUZZY RUNGE-KUTTA METHOD - (FRK-4)

In this section, for a fuzzy multiple retarded delay differential equation (1), we develop the fourth order Runge-Kutta method for multiple delay $f(t,y(t),y(t-\tau_1),y(t-\tau_2),\dots,y(t-\tau_n))$ by an application of Runge-Kutta method for fuzzy differential equation when *f* in (1) can be obtained via the Zadeh extension principle from. $f\in C[R^{+}\times R\times\ldots\times R,R]$. We are using Runge-Kutta method for multiple delay to solve the system of the differential equation because of its stability was already discussed by Guang DaHu, Gurang Di Hu and S.A. Meguid [4]. We assume that the existence and uniqueness of solutions of (1) hold for each $[t_{k,}t_{k+1}]$. The following method is the extension of [5]. The

Runge Kutta method is the fourth order approximation of $\underline{Y}_k^{'}(t,\alpha)$ and $\overline{Y}_k^{'}(t,\alpha)$.

We define

$$\underline{y}(t_{n+1};\alpha)-\underline{y}(t_n;\alpha)=\sum_{i=1}^{4} w_i\,\underline{K}_i(t_n;y(t_n;\alpha)),$$

$$\overline{y}(t_{n+1};\alpha)-\overline{y}(t_n;\alpha)=\sum_{i=1}^{4} w_i\,\overline{K}_i(t_n;y(t_n;\alpha)).$$

where w_1, w_2, w_3 and w_4 are constants and

$$\begin{aligned}
\underline{K}_1(t;y(t;\alpha)) = \min\{hg(t, y(t),\ & y(t-\tau_1),\dots,y(t-\tau_2)) \mid \\
& y(t)\in[\underline{y}(t_{k,n};\alpha),\overline{y}(t_{k,n};\alpha)],\\
& y(t-\tau_1)\in[\underline{y}(t_{k,n}-\tau_1;\alpha),\overline{y}(t_{k,n}-\tau_1;\alpha)],\dots,\\
& y(t-\tau_n)\in[\underline{y}(t_{k,n}-\tau_n;\alpha),\overline{y}(t_{k,n}-\tau_2;\alpha)]\}
\end{aligned}$$

$$\begin{aligned}
\overline{K}_1(t;y(t;\alpha)) = \max\{hg(t, y(t),\ & y(t-\tau_1),\dots,y(t-\tau_n)) \mid \\
& y(t)\in[\underline{y}(t_{k,n};\alpha),\overline{y}(t_{k,n};\alpha)],\\
& y(t-\tau_1)\in[\underline{y}(t_{k,n}-\tau_1;\alpha),\overline{y}(t_{k,n}-\tau_1;\alpha)],\dots,\\
& y(t-\tau_n)\in[\underline{\phi}(t_{k,n}-\tau;\alpha),\overline{\phi}(t_{k,n}-\tau;\alpha)]\}
\end{aligned}$$

$$\begin{aligned}
\underline{K}_2(t;y(t;\alpha)) = \min\{hg(t+\tfrac{h}{2}, y(t),\ & y(t-\tau_1),\dots,y(t-\tau_n)) \mid \\
& y(t)\in[\underline{z}_1(t_{k,n},\underline{y}(t_{k,n};\alpha)),\overline{z}_1(t_{k,n},\overline{y}(t_{k,n};\alpha))],\\
& y(t-\tau_1)\in[\underline{z}_1(t_{k,n}-\tau_1,\underline{y}(t_{k,n}-\tau_1;\alpha)),\\
& \qquad \overline{z}_1(t_{k,n}-\tau_1,\overline{y}(t_{k,n}-\tau_1;\alpha))],\dots,\\
& y(t-\tau_n)\in[\underline{z}_1(t_{k,n}-\tau_n,\underline{y}(t_{k,n}-\tau_n;\alpha)),\\
& \qquad \overline{z}_1(t_{k,n}-\tau_n,\overline{y}(t_{k,n}-\tau_n;\alpha))]\}
\end{aligned}$$

$$\begin{aligned}
\overline{K}_2(t;\phi(t;\alpha)) = \max\{hg(t+\tfrac{h}{2}, y(t),\ & y(t-\tau_1),\dots,y(t-\tau_n)) \mid \\
& y(t)\in[\underline{z}_1(t_{k,n},\underline{y}(t_{k,n};\alpha)),\overline{z}_1(t_{k,n},\overline{y}(t_{k,n};\alpha))],\\
& y(t-\tau_1)\in[\underline{z}_1(t_{k,n}-\tau_1,\underline{y}(t_{k,n}-\tau_1;\alpha)),\\
& \qquad \overline{z}_1(t_{k,n}-\tau_1,\overline{y}(t_{k,n}-\tau_1;\alpha))],\dots,\\
& y(t-\tau_n)\in[\underline{z}_1(t_{k,n}-\tau_n,\underline{y}(t_{k,n}-\tau_n;\alpha)),\\
& \qquad \overline{z}_1(t_{k,n}-\tau_n,\overline{y}(t_{k,n}-\tau_n;\alpha))]\}
\end{aligned}$$

$$\begin{aligned}
\underline{K}_3(t;y(t;\alpha)) = \min\{hg(t+\tfrac{h}{2}, y(t),\ & y(t-\tau_1),\dots,y(t-\tau_n)) \mid \\
& y(t)\in[\underline{z}_2(t_{k,n},\underline{y}(t_{k,n};\alpha)),\overline{z}_2(t_{k,n},\overline{y}(t_{k,n};\alpha))],\\
& y(t-\tau_1)\in[\underline{z}_2(t_{k,n}-\tau_1,\underline{y}(t_{k,n}-\tau_1;\alpha)),\\
& \qquad \overline{z}_2(t_{k,n}-\tau_1,\overline{y}(t_{k,n}-\tau_1;\alpha))],\dots,\\
& y(t-\tau_n)\in[\underline{z}_2(t_{k,n}-\tau_n,\underline{y}(t_{k,n}-\tau_n;\alpha)),\\
& \qquad \overline{z}_2(t_{k,n}-\tau_n,\overline{y}(t_{k,n}-\tau_n;\alpha))]\}
\end{aligned}$$

$$\begin{aligned}
\overline{K}_3(t;y(t;\alpha)) = \max\{hg(t+\tfrac{h}{2}, y(t),\ & y(t-\tau_1),\dots,y(t-\tau_n)) \mid \\
& y(t)\in[\underline{z}_2(t_{k,n},\underline{y}(t_{k,n};\alpha)),\overline{z}_2(t_{k,n},\overline{y}(t_{k,n};\alpha))],\\
& y(t-\tau_1)\in[\underline{z}_2(t_{k,n}-\tau,\underline{y}(t_{k,n}-\tau_1;\alpha)),\\
& \qquad \overline{z}_2(t_{k,n}-\tau_1,\overline{y}(t_{k,n}-\tau_1;\alpha))],\dots,\\
& y(t-\tau_n)\in[\underline{z}_2(t_{k,n}-\tau,\underline{y}(t_{k,n}-\tau_n;\alpha)),\\
& \qquad \overline{z}_2(t_{k,n}-\tau_n,\overline{y}(t_{k,n}-\tau_n;\alpha))]\}
\end{aligned}$$

$$\begin{aligned}
\underline{K}_4(t;\phi(t;\alpha)) = \min\{hg(t+\tfrac{h}{2}, y(t),\ & y(t-\tau),\dots,y(t-\tau)) \mid \\
& y(t)\in[\underline{z}_2(t_{k,n},\underline{y}(t_{k,n};\alpha)),\overline{z}_3(t_{k,n},\overline{y}(t_{k,n};\alpha))],\\
& y(t-\tau_1)\in[\underline{z}_3(t_{k,n}-\tau_1,\underline{\phi}(t_{k,n}-\tau_1;\alpha)),\\
& \qquad \overline{z}_3(t_{k,n}-\tau_1,\overline{\phi}(t_{k,n}-\tau_1;\alpha))],\dots,\\
& y(t-\tau_n)\in[\underline{z}_3(t_{k,n}-\tau_n,\underline{\phi}(t_{k,n}-\tau_n;\alpha)),\\
& \qquad \overline{z}_3(t_{k,n}-\tau_n,\overline{\phi}(t_{k,n}-\tau_n;\alpha))]\}
\end{aligned}$$

$$\begin{aligned}
\overline{K}_4(t;y(t;\alpha)) = \max\{hg(t+\tfrac{h}{2}, y(t),\ & y(t-\tau),\dots,y(t-\tau_n)) \mid \\
& y(t)\in[\underline{z}_3(t_{k,n},\underline{y}(t_{k,n};\alpha)),\overline{z}_3(t_{k,n},\overline{y}(t_{k,n};\alpha))],\\
& y(t-\tau_1)\in[\underline{z}_3(t_{k,n}-\tau_1,\underline{y}(t_{k,n}-\tau_1;\alpha)),\\
& \qquad \overline{z}_3(t_{k,n}-\tau_1,\overline{y}(t_{k,n}-\tau_1;\alpha))],\dots,\\
& y(t-\tau_n)\in[\underline{z}_3(t_{k,n}-\tau_n,\underline{y}(t_{k,n}-\tau_n;\alpha)),\\
& \qquad \overline{z}_3(t_{k,n}-\tau_n,\overline{y}(t_{k,n}-\tau_n;\alpha))]\}
\end{aligned}$$

Next we define,

$$\underline{z}_1(t_{k,n},\underline{y}(t_{k,n};\alpha)) = \underline{y}(t_{k,n};\alpha) + \frac{1}{2}\underline{K}_1(t_{k,n},\underline{y}(t_{k,n};\alpha)),$$

$$\overline{z}_1(t_{k,n},\overline{y}(t_{k,n};\alpha)) = \overline{y}(t_{k,n};\alpha) + \frac{1}{2}\overline{K}_1(t_{k,n},\overline{y}(t_{k,n};\alpha)),$$

$$\underline{z}_2(t_{k,n},\underline{y}(t_{k,n};\alpha)) = \underline{y}(t_{k,n};\alpha) + \frac{1}{2}\underline{K}_2(t_{k,n},\underline{y}(t_{k,n};\alpha)),$$

$$\overline{z}_2(t_{k,n},\overline{y}(t_{k,n};\alpha)) = \overline{y}(t_{k,n};\alpha) + \frac{1}{2}\overline{K}_2(t_{k,n},\overline{y}(t_{k,n};\alpha)),$$

$$\underline{z}_3(t_{k,n},\underline{y}(t_{k,n};\alpha)) = \underline{y}(t_{k,n};\alpha) + \frac{1}{2}\underline{K}_3(t_{k,n},\underline{y}(t_{k,n};\alpha)),$$

$$\overline{z}_3(t_{k,n},\overline{y}(t_{k,n};\alpha)) = \overline{y}(t_{k,n};\alpha) + \frac{1}{2}\overline{K}_3(t_{k,n},\overline{y}(t_{k,n};\alpha)),$$

Then we define

$$P[(t,\underline{y}(t;\alpha),\overline{y}(t;\alpha)] = \underline{K}_1(t;y(t;\alpha)) + 2\underline{K}_2(t;y(t;\alpha)) + 2\underline{K}_3(t;y(t;\alpha)) + \underline{K}_4(t;y(t;\alpha)),$$

$$Q[(t,\underline{y}(t;\alpha),\overline{y}(t;\alpha)] = \overline{K}_1(t;y(t;\alpha)) + 2\overline{K}_2(t;y(t;\alpha)) + 2\overline{K}_3(t;y(t;\alpha)) + \overline{K}_4(t;y(t;\alpha)).$$

The approximate solution (3) is given by

$$\begin{cases} \underline{y}(t_{n+1};\alpha) = \underline{y}(t_n;\alpha) + \underline{y}(t_n-\tau;\alpha) + \dfrac{1}{6}P[(t_n,\underline{y}(t_n;\alpha),\underline{y}_n(t;\alpha))], \\ \overline{y}(t_{n+1};\alpha) = y(t_n;\alpha) + \underline{y}(t_n-\tau;\alpha) + \dfrac{1}{6}Q[(t_n,\overline{y}(t_n;\alpha),\overline{y}_n(t;\alpha))] \end{cases} \quad (6)$$

The approximate solution (5) is given by

$$\begin{cases} \underline{y}(t_{n+1};\alpha) = \underline{y}(t_n-\tau;\alpha) + \dfrac{1}{6}P[(t_n,\underline{y}(t_n;\alpha),\underline{y}_n(t;\alpha))], \\ \overline{y}(t_{n+1};\alpha) = \underline{y}(t_n-\tau;\alpha) + \dfrac{1}{6}Q[(t_n,\overline{y}(t_n;\alpha),\overline{y}_n(t;\alpha))] \end{cases} \quad (7)$$

NUMERICAL EXAMPLE

In this section we apply the FRK-4 of FMRDDE to FPMRDDE.

Consider the following fuzzy pure multiple retarded delay differential equation

$$\begin{cases} y'(t;\alpha) = (0.75+0.25\alpha) \\ \qquad y(t-1)+y(t-2)+y(t-3), \\ \qquad (1.125-0.125\alpha) \\ \qquad y(t-1)+y(t-2)+y(t-3), & 0 \le t \le 5 \\ y(t;\alpha) = (0.75+0.25\alpha)t^2, \\ \qquad (1.125-0.125\alpha)t^2, & -3 \le t \le 0 \end{cases} \quad (8)$$

The exact solution is given by

$$Y(t;\alpha) = (0.75+0.25\alpha)Y(t), \quad (1.125-0.125\alpha)Y(t), \quad 0 \le t \le 5, \quad 0 \le \alpha \le 1 \quad (9)$$

where,

$$Y(t) = \begin{cases} t^2, & (t<0) \\ 14t-6t^2+t^3, & (0 \le t < 1) \\ 1/12(115-96t+114t^2 - 28t^3+3t^4), & (1 \le t < 2) \\ 1/60(1839-1280t-270t^2 + 140t^3-354t^4+3t^5), & (2 \le t < 3) \\ 1/120(1650+2012t-1545t^2 + 320t^3+55t^4-14t^5+t^6), & (3 \le t \le 4) \\ 1/2520(-871015+620172t - 120477t^2-280t^3+1995t^4 + 168t^5-49t^6+3t^7), & (4 \le t < 5) \end{cases}$$

The exact solution (9) and the approximate solution obtained by using above FRK-4 method is given in Table:2 and in Table 1 respectively and plotted in Figure:1 and 2.

Table:1
Approximate Values of FPMRDDE.

t	Approximate value $\underline{y}(t;\alpha)$	Approximate value $\overline{y}(t;\alpha)$
0	126.055060427564	189.082590641346
0.1	130.256895775150	186.981672967553
0.2	134.458731122735	184.880755293761
0.3	138.660566470320	182.779837619968
0.4	142.862401817906	180.678919946175
0.5	147.064237165491	178.578002272382
0.6	151.266072513077	176.477084598590
0.7	155.467907860662	174.376166924797
0.8	159.669743208248	172.275249251004
0.9	163.871578555833	170.174331577211
1.0	168.073413903419	168.073413903419

Table:2
Exact Value of FPMRDDE.

t	Exact value $\underline{Y}(t;\alpha)$	Exact value $\overline{Y}(t;\alpha)$
0	126.055059523810	189.082589285714
0.1	130.256894841270	186.981671626984
0.2	134.458730158730	184.880753968254
0.3	138.660565476190	182.779836309524
0.4	142.862400793651	180.678918650794
0.5	147.064236111111	178.578000992064
0.6	151.266071428571	176.477083333333
0.7	155.467906746032	174.376165674603
0.8	159.669742063492	172.275248015873
0.9	163.871577380952	170.174330357143
1.0	168.073412698413	168.073412698413

Approximate and Exact Value of FPMRDDE.

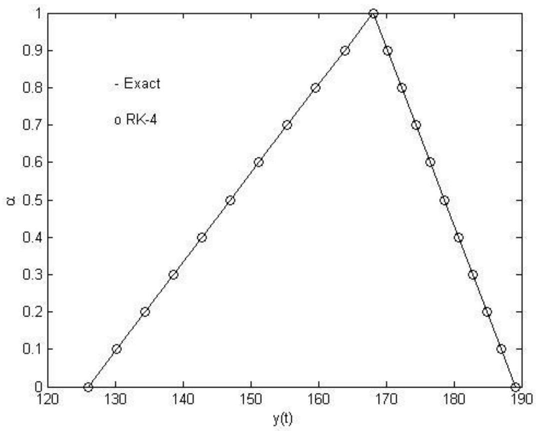

Figure:1 For $\alpha \in [0,1], t = 5$

Approximate and Exact Value of FPMRDDE

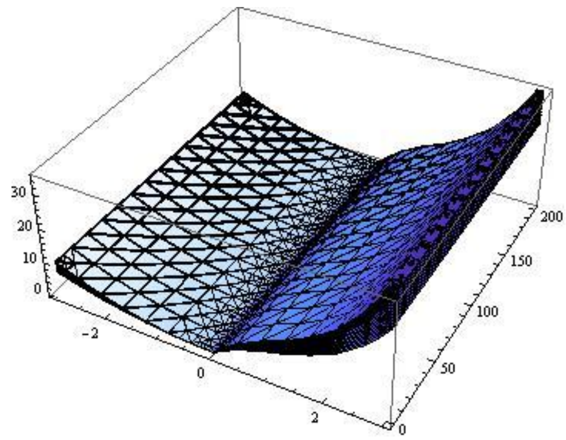

Figure:2 For $\alpha \in [0,1], t \in [0,5]$

5. CONCLUSION

In this article, we have developed numerical solution of Fuzzy multiple retarded delay differential equations and we applied the same method to solve Fuzzy Pure multiple retarded delay differential equations. The approximate solution coincides with the exact solutions as shown in the figure1 and in figure 2. Thus the Fuzzy Runge-Kutta method of order four holds good to solve any kind of multiple retarded delay differential equations as well as Fuzzy Pure Multiple retarded delay differential equations. So it can be implemented to any such pure multiple retarded delay differential equations modeling physical problems or the replica of the problems also. From the Table:1 the accuracy of the method is about four decimal places.

Conflict of interest

The author confirms that there is no conflict of interest to declare for this publication.

Acknowledgement

The author would like to express his sincere thanks to the referees for their valuable suggestions towards the improvement of the paper.

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